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Observed Natural Frequency Pile

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Observed and Predicted Natural Frequency of a Pile Foundation

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SYNOPSIS: Vertical and horizontal vibration tests were conducted on a 45 cm. diameter concrete driven pile. The pile length was 17 m. The response of the pile was computed following the commonly used approach of Novak (1974, 1977) and Novak and El-Sharnouby (1983). A comparison was made of the predicted and observed response, the results of which are discussed in the paper.

INTRODUCTION

Piles are used to support structures in a variety of situations involving both static and dynamic loads. Piles are used as foundations for machines and other vibrating equipment when operating conditions of the machine limit the vibration amplitudes to very small values and usual block type foundations are not feasible. A pile foundation supporting a machine may be excited in vertical, horizontal, rocking or torsional modes of vibration depending upon the nature of unbalanced forces generated by the machine. The response of a pile supported machine foundation is generally obtained by using one of the simplified approaches such as (1) using the concept of elastic subgrade reaction (Barkan [1962], Maxwell et al. [1969]), for obtaining equivalent soil springs, (2) treating the pile as a cantilever fixed at the lower end, (3) treating the pile problem as a case of one dimensional wave propagation in a rod (Richart, Hall and Woods [1970]), and (4) extending the solution of Baranov (1967) for embedded foundations and determining the stiffness and damping of the soil-pile system from the elastic half space approach (Novak [1974, 1977], Novak and El-Sharnouby [1983], Novak and Howell [1977]).

Well designed machine foundations are characterized by small vibration amplitudes. Therefore, linear theories seem adequate for calculating the response of such foundations. The theories of Novak (1974, 1977), and Novak and El-Sharnouby (1983) have been commonly used for this purpose. Very little data comparing the predicted and the observed pile response is presently available. In this paper an attempt has been made to compare the observed response of a full scale pile with its calculated response using the approach of Novak (1974, 1977), Novak and El-Sharnouby (1983) and Prakash and Puri (1988). The cases of constant shear modulus with depth (homogeneous soil profile) and the parabolic shear modulus variation with depth (parabolic soil profile) have been considered. The results of this comparison are discussed in this paper.

FIELD TESTS ON PILE

Forced horizontal and vertical vibration tests were conducted on a 45 cm. diameter pile driven 17 m. into a deposit of clayey silt (Puri et al., 1977). A reinforced concrete cap 1.2 m. x 1.2 m. x 0.8 m. high was cast monolithically with the pile head for mounting the vibration generating equipment. The vibrations were monitored with the help of acceleration transducers mounted on the pile at mud line. The output from the acceleration transducers was amplified using universal amplifiers and recorded on ink writing oscillographs (strip chart recorder). A typical amplitude versus frequency plot for one of these tests is shown in Fig. 1.

Free horizontal vibration tests were also conducted on this pile by pulling and suddenly releasing. A typical free vibration record is shown in Fig. 2. The values of observed natural frequencies are shown in Table 1. The values of dynamic shear modulus at the site were determined by conducting block vibration, wave propagation and standard penetration tests. The data of these tests was interpreted following the approach suggested by Prakash and Puri (1982, 1988). The details of the tests for dynamic shear modulus determination are not discussed in this paper. The value of dynamic shear modulus at the level of pile tip was determined to be 650 kg/cm^2.

COMPUTED RESPONSE OF THE PILE

Soil and Pile Properties

Diameter of the pile d = 45.0 cm.
Embedded length l = 17.0 m.
Young's modulus of concrete $E_p = 2.2 \times 10^5\ kg/cm^2$

The weight of pile cap and pile = 2686.5 kg above the mud line
Mass moment of inertia of pile cap and pile (above mud line) about the horizontal axis of vibration M_m = 4686.4 kg cm/s^2
Dynamic Shear Modulus of Soil at the level of pile tip G_s = 650 kg/cm^2

Natural Frequency of Vertical Vibrations

The natural frequency of the pile in vertical vibrations was calculated from Eq. 1.

$$f_{nz} = \frac{1}{2\pi}\sqrt{\frac{k_z}{m}} \quad (1)$$

in which

f_{nz} = Natural frequency of vertical vibrations, Hz

k_z = Equivalent spring for the soil-pile system for vertical vibrations

and m = Mass of the cap and pile above the mud line.

The value of k_z is obtained from Eq. 2 (Novak and El-Sharnouby (1977).

$$k_z = \frac{E_p A}{r_0} f_{w_1} \quad (2)$$

in which, E_p = Young's modulus of pile material

A = Area of cross-section of the pile

r_0 = Pile radius

and f_{w1} = Stiffness parameter.

The values of f_{w_1} for the case of homogeneous soil profile and parabolic soil profile are obtained from Fig. 3. The values of the natural frequency of vertical vibrations so calculated are shown in Table 1.

Natural Frequency and Amplitude of Horizontal Vibrations

The undamped natural frequencies of horizontal vibrations of the soil pile system were computed by treating it as a case of coupled rocking and sliding and using Eq. 3.,

$$\omega_{n1,2}^2 = \frac{1}{2}\left(\frac{k_x}{m} + \frac{k_\phi}{M_m}\right) \pm \sqrt{\frac{1}{4}\left(\frac{k_x}{m} - \frac{k_\phi}{M_m}\right)^2 + \frac{k_{x\phi}^2}{mM_m}} \quad (3)$$

in which, ω_{n1} and ω_{n2} are the two natural frequencies in coupled rocking and sliding.

k_x = Translation stiffness coefficient

k_ϕ = Rotational stiffness coefficient

$k_{x\phi}$ = Cross-stiffness coefficient

and, M_m = Mass moment of inertia of the pile and pile cap.

The values of k_x, k_ϕ and $k_{x\phi}$ are obtained following the approach of Novak and El-Sharnouby (1983) and Prakash and Puri (1988).

$$k_x = \frac{E_p I_p}{r_0^3} f_{x1} \quad (4a)$$

$$k_\phi = \frac{E_p I_p}{r_0} f_{\phi 1} \quad (4b)$$

$$k_{x\phi} = \frac{E_p I_p}{r_0^2} f_{x\phi 1} \quad (4c)$$

in which,

I_p = Moment of inertia of pile cross-section

and f_{x1}, $f_{\phi 1}$ and $f_{x\phi 1}$ are stiffness parameters that are obtained from Table 2.

From the calculated values of $\omega_{n1,2}$, the two natural frequencies $f_{n\,1,2}$ are obtained from Eq. 5.

$$f_{n\,1,2} = \frac{\omega_{n1,2}}{2\pi} \text{ Hz} \quad (5)$$

The values of $f_{n\,1,2}$ for the homogeneous soil profile and the parabolic soil profile are shown in Table 1 in which the natural frequency of free vibrations is also shown.

The amplitude of horizontal vibrations A_x at mud line was calculated for different operating frequencies in the range 7 to 16 Hz by using Eq. 6 (Beredugo and Novak, 1972).

$$A_x = P_x \sqrt{\frac{\alpha_1^2 + \alpha_2^2}{\epsilon_1^2 + \epsilon_2^2}} \quad (6)$$

in which P_x = Horizontal exciting force.

The values of α_1, α_2, ϵ_1 and ϵ_2 for use in Eq. 6 are obtained as follows.

$$\alpha_1 = (k_\phi - M_m\omega^2 - \left(\frac{M_y}{P_x}\right)k_{x\phi} \quad (7a)$$

$$\alpha_2 = \left(c_\phi - \frac{M_y}{P_x}c_{x\phi}\right)\omega \quad (7b)$$

$$\epsilon_1 = mM_m\omega^4 - [mk_\phi + M_m k_x + c_x c_\phi - c_{x\phi}^2]\omega^2 + [k_x k_\phi - k_{x\phi}^2] \quad (7c)$$

$$\epsilon_2 = -[mc_\phi + M_m c_x]\omega^3 + [c_x k_\phi + c_\phi k_x - 2c_{x\phi}k_{x\phi}]\omega \quad (7d)$$

in which,

ω = Operating speed in

c_x = Translational damping constant

c_ϕ = Rotational damping constant

and $c_{x\phi}$ = Cross damping constant

The values of c_x, c_ϕ and $c_{x\phi}$ are obtained as follows (Novak and El Sharnouby, 1983).

$$c_x = \frac{E_p I_p}{r_0^2 V_s} f_{x2} \quad (8a)$$

$$c_\phi = \frac{E_p I_p}{V_s} f_{\phi 2} \quad (8b)$$

$$c_{x\phi} = \frac{E_p I_p}{r_0 V_s} f_{x\phi 2} \quad (8c)$$

in which V_s = Shear wave-velocity in soil,

and f_{x2}, $f_{\phi 2}$ and $f_{x\phi 2}$ are damping parameters that are obtained from Table 2.

The values of horizontal amplitude A_x were calculated for the cases of homogeneous and parabolic soil profile.

The calculated values of peak (resonant) horizontal amplitudes for the soil pile system for all cases are given in Table 1. The calculated values of horizontal amplitudes at different frequencies have been plotted in Fig. 1.

DISCUSSION AND CONCLUSIONS

1. The computed natural frequencies of vertical vibrations of the pile for the homogeneous and parabolic soil profiles are 46.0 and 38.8 Hz respectively (Table 1). The observed natural frequency of vertical vibrations is 32.2 Hz. The computed values of natural frequency for the homogeneous soil profile is 43% higher than the observed natural frequency of vertical vibrations. For the parabolic soil profile, the calculated natural frequency is 20.5% larger than the observed natural frequency.
2. For the case of coupled rocking and sliding, the calculated values of smaller natural frequency f_{n1} are 30.9

ıd 11.89 Hz for the homogeneous and parabolic soil cofiles respectively (Table 1). The observed natural cequency is 10.3 Hz. The calculated natural frequency or the uniform soil profile is substantially higher than ıe observed natural frequency of horizontal vibrations. or the case of parabolic soil profile the calculated atural frequency is about 15% higher than the observed atural frequency.

. The computed natural frequency of horizontal free ibrations for the parabolic soil profile is 12.9 Hz and s 12% higher than the observed frequency of free ibrations (Table 1).

. The observed values of the peak horizontal vibration nplitude is 0.44 mm. which is higher than the calculated nplitudes for the homogeneous soil profile (0.08745 mm.) ıd the parabolic soil profile (0.116 mm.). In the freıency range considered (Fig. 1), the computed amplitudes f horizontal vibrations are generally smaller and near esonance they are substantially smaller than the bserved values.

. Because of the limited nature of the study, it is not ossible to draw any general conclusions, but it seems hat in this particular case the assumption of a parabol- soil profile has given reasonable values of natural requencies both for the case of vertical as well as orizontal vibrations.

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TABLE 1. Comparison of Observed and Computed Data

VIBRATION MODE	ITEM	OBSERVED VALUES		COMPUTED VALUES		
				UNIFORM SOIL PROFILE	PARABOLIC SOIL PROFILE	
		FORCED VIBRATIONS	FREE VIBRATIONS	FORCED VIBRATIONS	FORCED VIBRATIONS	FREE VIBRATIONS
VERTICAL	f_{nz} Hz	32.2	-	46.0	38.8	-
HORIZONTAL	f_{n1} Hz	10.3	11.5	30.9	11.84	12.9
	f_{n2} Hz	-	-	77.6	45.7	-
	A_x mm	0.44	-	0.08745	0.116	

Table 2. Stiffness and Damping Parameters for Horizontal Vibrations of Long Piles (Novak and El-Sharnouby, 1983)

ν (1)	E_{pile}/G_{soil} (2)	$f_{\phi 1}$ (3)	$f_{x\phi 1}$ (4)	f_{x1} (5)	f^{p}_{x1} (6)	$f_{\phi 2}$ (7)	$f_{x\phi 2}$ (8)	f_{x2} (9)	f^{p}_{x2} (10)
		Stiffness Parameters				Damping Parameters			
		(a) Homogeneous Soil Profile							
0.25	10,000	0.2135	−0.0217	0.0042	0.0021	0.1577	−0.0333	0.0107	0.0054
	2,500	0.2998	−0.0429	0.0119	0.0061	0.2152	−0.0646	0.0297	0.0154
	1,000	0.3741	−0.0668	0.0236	0.0123	0.2598	−0.0985	0.0579	0.0306
	500	0.4411	−0.0929	0.0395	0.0210	0.2953	−0.1337	0.0953	0.0514
	250	0.5186	−0.1281	0.0659	0.0358	0.3299	−0.1786	0.1556	0.0864
0.40	10,000	0.2207	−0.0232	0.0047	0.0024	0.1634	−0.0358	0.0119	0.0060
	2,500	0.3097	−0.0459	0.0132	0.0068	0.2224	−0.0692	0.0329	0.0171
	1,000	0.3860	−0.0714	0.0261	0.0136	0.2677	−0.1052	0.0641	0.0339
	500	0.4547	−0.0991	0.0436	0.0231	0.3034	−0.1425	0.1054	0.0570
	250	0.5336	−0.1365	0.0726	0.0394	0.3377	−0.1896	0.1717	0.0957
		(b) Parabolic Soil Profile							
0.25	10,000	0.1800	−0.0144	0.0019	0.0008	0.1450	−0.0252	0.0060	0.0028
	2,500	0.2452	−0.0267	0.0047	0.0020	0.2025	−0.0484	0.0159	0.0076
	1,000	0.3000	−0.0400	0.0086	0.0037	0.2499	−0.0737	0.0303	0.0147
	500	0.3489	−0.0543	0.0136	0.0059	0.2910	−0.1008	0.0491	0.0241
	250	0.4049	−0.0734	0.0215	0.0094	0.3361	−0.1370	0.0793	0.0398
0.40	10,000	0.1857	−0.0153	0.0020	0.0009	0.1508	−0.0271	0.0067	0.0031
	2,500	0.2529	−0.0284	0.0051	0.0022	0.2101	−0.0519	0.0177	0.0084
	1,000	0.3094	−0.0426	0.0094	0.0041	0.2589	−0.0790	0.0336	0.0163
	500	0.3596	−0.0577	0.0149	0.0065	0.3009	−0.1079	0.0544	0.0269
	250	0.4170	−0.0780	0.0236	0.0103	0.3468	−0.1461	0.0880	0.0443

f^{p}_{x1} and f^{p}_{x2} are for pinned end.

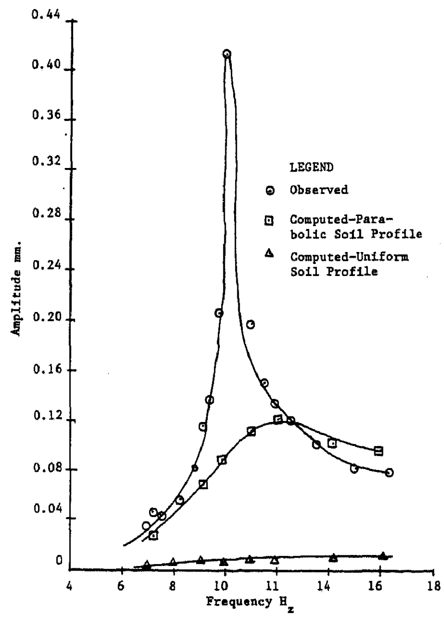

Fig. 1. Typical Amplitude vs. Frequency Plots

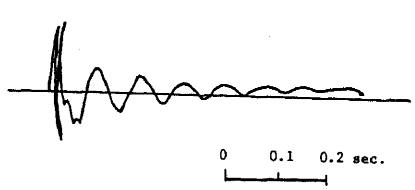

Fig. 2. Typical Free (Horizontal) Vibration Record

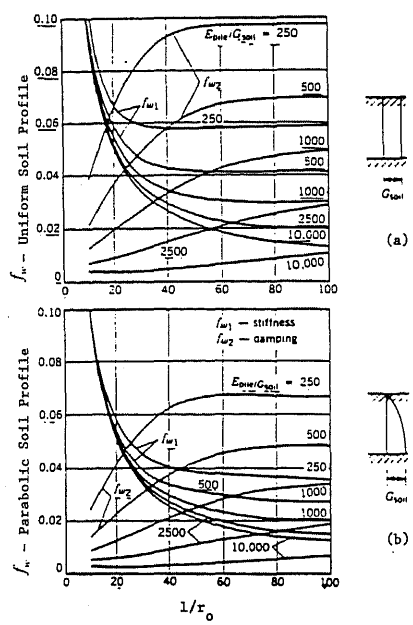

Fig. 3. Stiffness and Damping For Fixed-Tip Vertically Vibrating Piles (a) Homogeneous (b) Parabolic Soil Profiles (Novak and El-Sharnouby, 1983)